# Exercise I
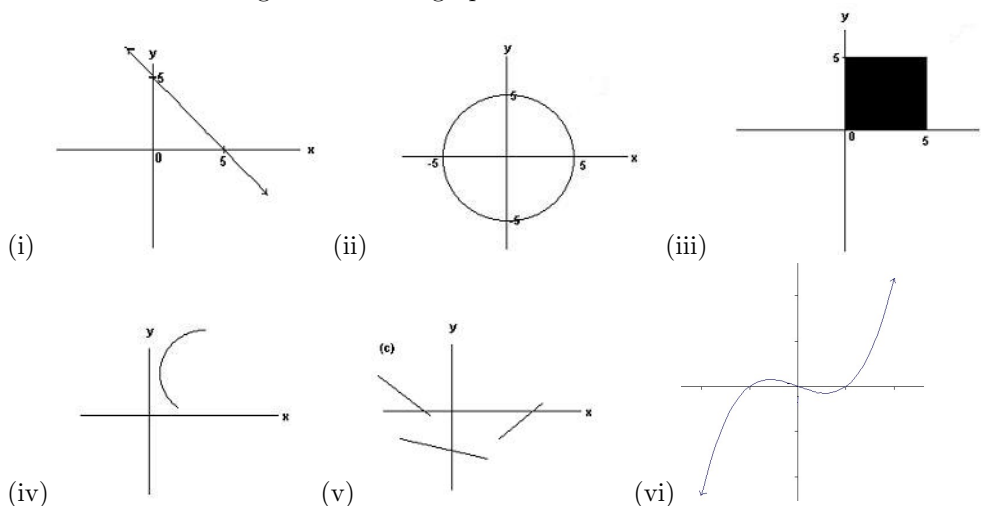
1. Sketch the graphs of each the following functions:

   (i) $x \mapsto 4 - x^2$   (ii) $y = x^2 - x - 6$;   (iii) $f(x) = x^2 - 2x - 8$;
   (iv) $y = (x-1)(x-2)(x-3)$;   (v) $y = (x+1)(x^2 - 5x + 6)$;
   (vi) $y = (x+1)(x^2 + x + 1)$;   (vii) $g(x) = \dfrac{1}{x-1}$;   (viii) $y = |x|$;
   (ix) $y = |x - 2|$;

2. Which of the following could be the graph of a function:

   

   (i)                    (ii)                    (iii)

   

   (iv)                    (v)                    (vi)

3. Let the function $f$ be defined by:

$$f(x) = \begin{cases} x + 1, & x \le -2, \\ x^2, & -2 < x \le 2 \\ 2x, & x > 2. \end{cases}$$

   (i) Find $f(1)$.   (ii) Find $f(3)$.   (iii) Find $f(-4)$.
   (iv) Sketch the graph of $f$.

4. Sketch the graph of

   (i) $\left\{ \dfrac{1}{n} \right\}$,   (ii) $\left\{ \left( \dfrac{-1}{2} \right)^n \right\}$.

5.  (i) Draw a graph of an increasing function which has domain $[1,5]$ and range $[2,4]$.

    (ii) Draw a graph of an increasing function $f(x)$ which has domain $[1,5]$ and range $[2,4]$ with $f''(x) < 0$ for all $x \in (1,5)$.

    (iii) Draw a graph of an increasing function $f(x)$ which has domain $[1,5]$ and range $[2,4]$ with $f''(x) > 0$ for all $x \in (1,5)$.

6. For each of the following functions $f$ compute the derivative $f'(x)$:
   (i) $f(x) = (x^2 + x)^2$           (ii) $f(x) = \sqrt[3]{x + 4x^5}$
   (iii) $f(x) = (x^3 + 1)^4 \sin(x^2 + 3)$   (iv) $f(x) = \dfrac{\cos(x^3 + x)}{\sin(x^2 + 1)}$